

# Working Principle of an EM Cancer Detector

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**Abstract.** C. Vedruccio invented an electronic system for non-invasive EM cancer detection, requiring only that a handy probe be moved a few centimetres over the surface of the body, close to the organ that should be tested. The probe contains a *non-linear* oscillator, emitting a very weak EM wave, with several frequency components (at 450, 900 and 1350 MHz, for instance). They are displayed on the screen of a spectrum analyser, which is fed by a small antenna that is situated about 2 meters away from the probe. The relative intensities of these spectral lines are predetermined, but when the probe is brought close to biological tissue, the height of one or several lines can be *strongly reduced*, according to the pathological state of the tested tissue. We explain this phenomenon, by establishing a mathematical model and by solving the resulting equations. Actually, the probe does stimulate minute electrical oscillations inside the tissue, but this requires an energy transfer, which is clearly detectable because of the peculiar properties of *non-linear resonance interaction*.

## Introduction

In March 2000, the author met *Clarbruno Vedruccio* in Italy and incidentally, he heard about his astonishing invention. He had developed an electronic system for the detection of non-metallic mines, but discovered that it could also detect pathological modifications inside the human body. It appeared even that this system is particularly useful and efficient for *early cancer detection*. Since the reasons for this quasi-magical capability were not clear and since the author had studied the interaction of EM waves with matter of various types, including biological tissues, he was very intrigued.

Having the opportunity to see this equipment and its performance, he was also astonished about the great *simplicity* of this new procedure. The human subject is standing, normally dressed, while a cylindrical probe is held a few centimetres away from the surface of the body, close to the organ that should be tested. The probe contains only batteries, an electrical circuit and a small antenna, situated inside a partially reflecting cavity. It emits a *very weak* EM wave that is not sinusoidal, but perfectly periodic, with a preset repetition frequency of 450 MHz, for instance. A small receiving antenna, separated from the probe by about 2 meters, feeds a spectrum analyser that displays equally spaced spectral lines that correspond to the fundamental frequency and at least two harmonics. The height of one or several of these spectral lines is drastically *reduced*, however, when the probe is brought close to some pathologically modified biological tissue. Normal tissue leaves the spectrum unaffected. Is it possible to explain such a strange behaviour?

The author wrote already in 2000 a report that had several objectives. (1) It reviewed and *explained the basic theory* of EM interactions with materials of various kinds, from kHz to GHz frequencies. Several processes are possible, indeed, but all of them can be treated in classical terms, since they don't involve discrete quantum-mechanical transitions. (2) The report presented also an overview of *experimental results for biological tissues*, obtained between 1920 and 1999. Actually, one measured the impedance of blood and various types of

excised tissues, introduced in specially designed cells, to determine the values of the dielectric constant  $\epsilon$  and the electrical conductivity  $\sigma$  at many frequencies. The technical tools were constantly improved and the spectral domain was progressively enlarged, but the measured values of  $\epsilon$  and  $\sigma$  could always be related to established theories. (3) It was shown that *the electrical properties of malignant tissues* display significant differences with respect to those of homologous normal tissues. EM cancer detection should thus be possible!

This report remained confidential, since the invention was not yet legally protected, but will be available on Internet<sup>1</sup> in a version that includes more recent results, presents new experimental methods and proposes some ideas to explain the difference between normal and malignant tissues. Basically, the oscillating electric field of the EM wave can set charged particles in forced oscillations, polarize neutral particles and membranes or produce rotations small particles that carry a permanent electric dipole. These responses can reveal that some changes occurred *at the molecular level*, because of pathological modifications, but all this knowledge remained useless for diagnostic purposes, as long as the necessary tests had to be performed by means of complicated *in vitro* measurements.

Some authors tried therefore to simplify this procedure. To get *in vivo* measurements, one can determine the effects of a current passing through a more or less extended part of the human body, by measuring the resulting potential difference near these points or further away<sup>2</sup>. This yields already useful results, but they are rather imprecise, since they depend on inhomogeneous tissue structures, situated between the surface electrodes. A more refined, but much more expensive method considers the reflection of microwaves<sup>3</sup>. This requires good contact for impedance matching and relatively high intensities, especially when malignant tissues should be destroyed, because of their higher specific absorption of microwave energy. Vedruccio's system is radically different<sup>4</sup>.

Contact with the body surface is not necessary and there are no health hazards. According to recent measurements, the receiving antenna of the spectrum analyser catches only 1.58  $\mu$ W, 100 nW and 1 nW, respectively for the first, second and third spectral line. Moreover, an examination doesn't have to last more than about 5 minutes. Although the "bioscanner" explores only what happens at three (or eventually four) frequencies, they can be chosen to optimise the discrimination of selected anomalies in particular organs. Rigorous medical tests have already been performed in different institutions, with the approval of ethical commissions<sup>5</sup>. We mention here only that *the presence, as well as the absence of malignant prostate cancer* could be established in more than 90% of the tested cases, with confirmations by biopsy or other methods. The instrument is under development by the Italian company Galileo-Alenia<sup>6</sup> and should be available in the fall of 2003, with computer software to facilitate diagnostic interpretation. Further medical tests are underway.

Although this instrument applies physical principles to visualize hidden realities, like a cardiograph or X-rays for instance, it is *new* and its capabilities are so extraordinary and unsuspected that scepticism is quite natural. Oncologists could even object that there are various forms of cancer cells and different stages of development. How could the usually necessary, very painstaking microscopic analyses be shortcut by an apparently very global method? First of all, the aim of the new development is not to replace these analytic methods, but to provide a simple and economic method for *preliminary screening*. If this could be realised early enough, it would definitely be advantageous. Moreover, we shouldn't give more credit to *a priori* judgements concerning the diagnostic efficiency and selectivity of this method than to careful and controlled testing. It seems worthwhile to find out, but to avoid the suspicion of charlatanism, it is also necessary to *clarify the working principle* of this apparatus. This is even a matter of normal scientific curiosity.

In the first part of this paper, we show that it is possible to *conceive a model* of the probe and the tested biological tissue, so that the relevant behaviour of this system will be described

by means of *only two variables, satisfying two coupled differential equations*. One of them is non-linear. It is of the van der Pol type, initially introduced to account for the properties of a triode that activates an oscillating electric circuit<sup>7</sup>. Today, a transistor or a tunnel diode<sup>8</sup> replaces the vacuum tube. In the second part, *we solve these equations* for the simplest possible case, where the generator is tuned in such a way that it delivers a practically sinusoidal signal. There is thus only one frequency component, but it appears already that the non-linear differential equation has several surprising consequences. In the third part, we treat the more general case, where the generator produces at least *three spectral lines*. This accounts for the observed facts. As far as we know, the problem of “non-linear resonance interaction” (NLRI) has not yet been treated.

## 1. The coupled active and passive oscillators

To understand the working principle of Vedruccio’s invention, we don’t have to know constructional details, but it is essential to make it mentally transparent, by considering an *equivalent circuit diagram*. The left part of figure 1 represents the probe and the right part the tested biological tissue, while the intermediate, interrupted lines indicate how the coupling is realized. The sizes give a visual impression of hierarchy. For element T (triode, transistor or transistor-tunnel diode hybrid circuit), we indicate only what is really essential. T is powered by batteries and the grid of the triode or base of the transistor is inductively coupled to the self  $L_1$  of the first oscillating circuit, to get positive feedback, but for us, it is only important that the current  $I$  passing through T is a *non-linear* function of the applied potential difference  $V$ .

The first oscillating circuit and element T constitute, together, an *active oscillator* that can create electrical oscillations inside the tested biological tissue. We consider only one *passive oscillator*, although there are several subsystems that could oscillate, but their mutual interactions are negligible in regard to instantaneous electrical responses. We can even imagine an oscillating electric circuit, where the self  $L_2$  and the capacity  $C_2$  determine the value of the resonance frequency, while the resistance  $R$  defines the energy loss. We indicate currents and potential differences that will allow us to describe the behaviour of this system.

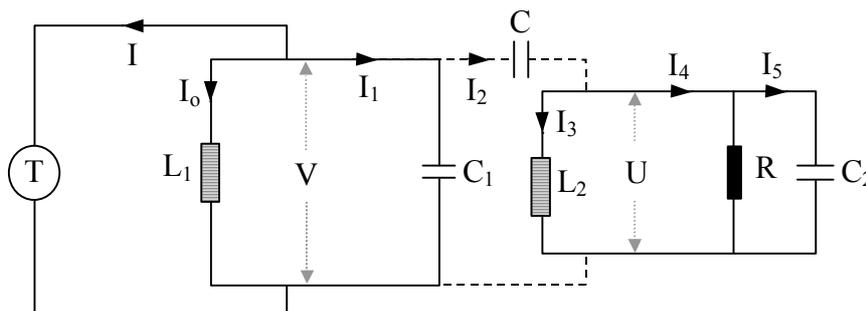


Figure 1: The equivalent electric circuit of the coupled active and passive oscillators.

First of all, we have to stress the fact that usually, one considers a magnetic coupling between oscillating circuits or an electric coupling where the capacity  $C$  is a common element of both circuits. Here, we consider a capacity  $C$  that allows an oscillatory current to pass from one circuit to the other.  $C$  increases when the probe is approaching the tissue. We have chosen this model, since we know that the probe contains a small antenna and that the tested biological tissue is only separated from the probe by a small distance compared to the wavelength, which is 65 cm at 460 MHz. This means that the passive oscillator is subjected to the “near field” of the emitted wave, where *retardation effects are negligible*. By creating a

standing wave, we could produce electrically equivalent situations at regularly distributed places, but here, we consider only the usual procedure, where the bioscanner emits a propagating wave. In the near field, the spatial distribution of the electrical field lines is then the same - at every particular instant - as for a static field (quasi-static approximation). The antenna plays thus in this region the role of an “open capacity”, but it creates an oscillating electric field. This field induces forced oscillations of charged particles, polarizes neutral particles or sets small dipolar particles in rotation, when the frequency is adequate.

It is important to note that the intrinsic properties of the biological tissue (represented by  $L_2$ ,  $C_2$  and  $R$ ) can't be affected by the coupling. The values of  $L_1$  and  $C_1$  are fixed by preliminary tuning. There are thus two distinct natural frequencies, determined by the products  $L_1C_1$  and  $L_2C_2$ . The resistance of the first circuit is assumed to be negligible or more precisely, to be compensated by the transistor  $T$ . It should be noted that a single parallel resistance is sufficient to characterize energy losses in the second circuit. Applying Kirchhoff's law, we get  $I_0 + I + I_1 = 0$  and  $I_2 = I_3 + I_4$ . Since these relations have to be satisfied at every particular instant, we can assert that the time derivatives

$$\dot{I}_0 + \dot{I} + \dot{I}_1 = 0 \quad \text{and} \quad \dot{I}_2 = \dot{I}_3 + \dot{I}_4$$

Actually, there are only two variables that have to be considered: the potential differences  $V$  and  $U$ . To show this, we note that in the first circuit,

$$V = L_1 \dot{I}_0 \quad \text{and} \quad I_1 - I_2 = \dot{Q}_1 = C_1 \dot{V} \quad \text{or} \quad \dot{I}_1 = C_1 \ddot{V} + C(\ddot{V} - \ddot{U}).$$

The current-voltage curve of element  $T$  is represented in figure 2. For small values of  $V$ , there appears a *negative resistance*. For larger values of  $V$ , the normal tendency, which yields increasing currents for increasing potential differences, is restored, but in general, we don't get a perfectly anti-symmetric curve.

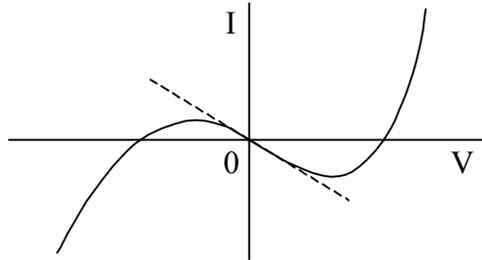


Figure 2: The current-voltage characteristic of element  $T$

A tunnel diode could be used alone<sup>9</sup>, but a more flexible design is preferable. We assume therefore that the negative resistance is modifiable and that

$$I = -\alpha V + \beta V^2 + \gamma V^3 \quad \text{so that} \quad \dot{I} = -(\alpha - 2\beta V - 3\gamma V^2)\dot{V}$$

Thus

$$C_1 \ddot{V} - (\alpha - 2\beta V - 3\gamma V^2)\dot{V} + \frac{V}{L_1} = C(\ddot{U} - \ddot{V}) \quad (1)$$

This equation is *non-linear*. When  $C = 0$ , it reduces to the famous *van der Pol equation*, initially introduced to explain the fact that triodes can be coupled to an oscillating circuit in such a way that one gets *oscillations of constant amplitude*. They are not necessarily

sinusoidal, but they are periodic and the repetition frequency is equal to the natural frequency of the oscillating circuit. We consider the more general case, where such an “auto-oscillator” is coupled to a passive oscillator. Equation (1) takes this into account by means of its second member. In the other circuit,

$$U = L_2 \dot{I}_3 = R(I_4 - I_5) \quad \text{and} \quad I_5 = \dot{Q}_2 = C_2 \dot{U}$$

It follows that

$$\frac{U}{L_2} + \frac{\dot{U}}{R} + C_2 \ddot{U} = C(\ddot{V} - \ddot{U}) \quad (2)$$

Equations (1) and (2) account for any possible behaviour of the active and passive oscillators. They should thus also help us to understand the surprising capabilities of the bioscanner, but it is useful to make these equations more transparent, by introducing natural units. We set  $\alpha - 2\beta V - 3\gamma V^2 = \alpha(1 - 2pmV - m^2V^2)$ , where  $m^2 = 3\gamma/\alpha$  and  $pm = \beta/\alpha$ . Then, we multiply (1) by  $m/C_1$  and (2) by  $m/C_2$ . This yields *two coupled differential equations* for  $x = mV$  and  $y = mU$ , where  $\omega_1^2 = 1/L_1C_1$  and  $\omega_2^2 = 1/L_2C_2$ , while  $\mu = \alpha/C_1$  and  $\nu = 1/RC_2$ :

$$\ddot{x} - \mu(1 - 2px - x^2)\dot{x} + \omega_1^2 x = k_1(\ddot{y} - \ddot{x}) \quad (3)$$

$$\ddot{y} + \nu\dot{y} + \omega_2^2 y = k_2(\ddot{x} - \ddot{y}) \quad (4)$$

The coupling constants  $k_1 = C/C_1$  and  $k_2 = C/C_2$  are pure, positive numbers, while  $\mu$  and  $\nu$  have the dimensions of a frequency. Equation (3) is *non-linear*. This will turn out to be very important but the most unusual feature is that the coupling involves second order time derivatives of the variables  $x$  and  $y$ . This *dynamic* coupling accounts for the fact that the capacity  $C$  facilitates the passage of high frequency currents. It is usually assumed that  $p = 0$ , but we will see that it is important to consider the more general case where  $p \neq 0$ .

To get a feeling for the solutions of (3) and (4), we consider some special cases. When  $C = 0$ , so that  $k_1 = k_2 = 0$ , we get *two distinct and well-separated oscillators*. The second one is simply a damped harmonic oscillator, while the first oscillator would be an ideal harmonic oscillator for the particular where  $\mu = 0$ . The general solution could then be written in the form  $x = A\cos(\omega t + \phi)$ , where  $\omega = \omega_1$ , while the amplitude  $A$  and the phase factor  $\phi$  are determined by initial conditions. Such an oscillator would have to be set in motion, indeed, by some specific action. As soon as  $\mu \neq 0$ , there appears a *qualitative change*, since the resting state  $x = 0$  becomes unstable. The slightest perturbation will automatically be amplified, but the capacity tends to be discharged. When  $\mu \ll 1$ , we get slowly amplified oscillations, but finally, the system will reach a *stationary state* of perfectly sustained, practically harmonic oscillations.

Larger values of  $\mu$  (for instance  $\mu = 1$  or  $\mu = 10$ ) will lead to rapid amplifications and to rapid discharges. A short time interval is then sufficient to reach the stationary state and we get a periodic succession of identical pulses. This is equivalent to a *superposition of harmonic waves*, with frequencies that are integer multiples of the fundamental frequency. Balthasar van der Pol investigated their waveforms and calculated “limit cycles”, represented by closed graphs in the  $(x, \dot{x})$  plane<sup>10</sup>. He called such a regular or quasi-regular succession of pulses a “relaxation oscillation” and he showed that it appears quite often in nature and technology. Cracking doors, waving flags, the beating heart and economic crises, provide good

examples<sup>11</sup>. But when the capacity  $C \neq 0$ , the differential equations (3) and (4) are coupled to one another and this gives rise to surprising phenomena.

## 2. Resonance interaction for one available frequency

To acquire *physical insight*, we start with the simplest possible situation, where  $\mu$  is very small, since the unperturbed generator would then produce a nearly harmonic oscillation. van der Pol considered already a very similar problem<sup>12</sup>, but his oscillating circuits were purely electronic ones, where the coupling capacity was a *common element* of both circuits. Since the capacity  $C$  is then respectively in series with  $C_1$  and  $C_2$ , the natural frequencies  $\omega_1$  and  $\omega_2$  that appear in equations (3) and (4) are modified. Moreover, the second members of (3) and (4) would then have to be replaced by  $-k_1\omega_1^2 y$  and  $-k_2\omega_2^2 x$ . B. van der Pol found that such a system has very remarkable properties, but he *missed* the phenomenon, which is central for EM cancer detection.

This results from the fact that his attention was focused on another phenomenon, related to the existence of *two possible frequencies*. They appeared already for an isolated unperturbed generator, when the current-voltage curve of element T contains a term in  $V^5$ . The system will then automatically choose one of the two possible frequencies<sup>13</sup>. When a resistance, included in the circuit, is progressively increased or decreased, the system tends always to preserve its previous state of oscillation, but this is only possible up to a certain point. We get thus a typical *hysteresis* phenomenon - which simply means: *delayed evolution*. Experimentally, van der Pol had observed already a similar phenomenon in 1920, when a triode oscillator was magnetically coupled to a passive oscillator, but this was published somewhat later<sup>10</sup>.

Appleton<sup>14</sup> considered then the case where *two triode oscillators* are magnetically coupled to one another. In ordinary, linear physics, it is well known that the coupling of two harmonic oscillators can lead to coordinated oscillations at the same frequency, but there are two possible frequencies  $\omega_{\pm}$ . They differ from the natural frequencies  $\omega_1$  and  $\omega_2$  of the uncoupled oscillators. Even for two identical, elastically coupled pendulums, they are slightly different from one another. In general, one gets then a *superposition* of these two “normal modes” of oscillation. The ratio depends on the initial conditions, but the superposition leads to beat phenomena. When two non-linear oscillators are coupled to one another, there are also two possible frequencies, but no beats when  $\omega_1$  and  $\omega_2$  are sufficiently close to one another, without being identical. The system chooses one of the possible frequencies.

Such a “*spontaneous synchronization*” had already been noted for pendulum clocks and organ pipes, but was explained for the first time by solving two coupled non-linear differential equations of the van der Pol type. Modifying  $\omega_2$  so that this value is approaching  $\omega_1$ , both oscillators will suddenly start to oscillate at the same frequency, although  $\omega_1$  and  $\omega_2$  are still different from one another. This will continue when  $\omega_2$  is increased, until the difference between  $\omega_2$  and  $\omega_1$  reaches the same absolute value as for the previous synchronization. This phenomenon is also called “*entrainment*”. It occurs even for very small amplitudes and has particularly important applications in the domain of biorhythms<sup>15</sup>. The circadian rhythm is entrained by the day and night variations of solar light and other rhythms are entrained by seasonal variations.

Before we solve the coupled differential equations (3) and (4), we recall the results of van der Pol’s experiment, performed in 1920. These results are summarized in figure 3 (redrawn from reference 12, 1922). The intensity  $x^2$  of the oscillations of the generator decreases when the natural frequency  $\omega_2$  of the passive oscillator is progressively increased, until it reaches a certain value, situated beyond the resonance frequency. At this point, the intensity of the

stationary oscillations of the generator “jumps” to a higher value. It is not yet the normal value for the unperturbed oscillator, but this value will finally be reached by a continued increase of  $\omega_2$ . When  $\omega_2$  is then decreased, the intensity of the stationary oscillations of the generator is also decreased. The process is symmetric, but not reversible in exactly the same way. There are thus two possible states of oscillation, but the system will automatically choose one of them. Since this state depends on past history, the coupled system displays *hysteresis*.

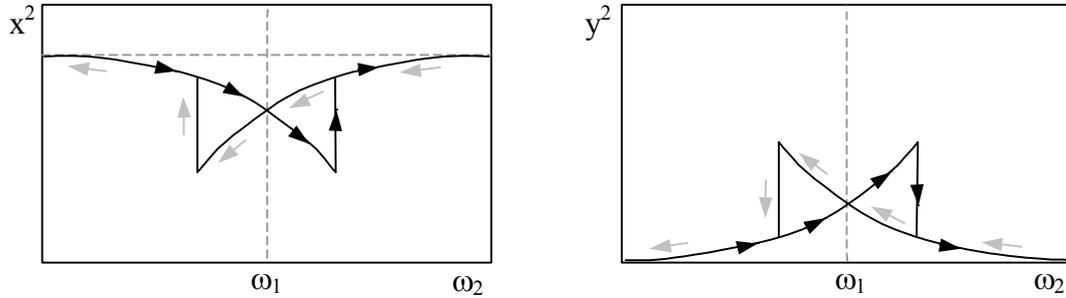


Figure 3: Variations of the intensities of the oscillations of the active and passive oscillators when the natural frequency of the passive oscillator is progressively increased or decreased.

The second part of figure 3 shows that the amplitude of the oscillations of the passive oscillator increases when those of the active oscillator decrease. This means that there is *an energy transfer*, but it cannot alternate, as this happens in beat phenomena. *van der Pol* could relate this to the non-linearity of equation (3). Actually, he combined the second order differential equations for  $x$  and  $y$ , to get a fourth order differential equation for  $x$ . It allowed for two frequencies  $\omega_{\pm}$  when  $\mu \ll 1$ . He considered therefore a superposition of oscillations at both frequencies, where the amplitudes can vary in the course of time:

$$x = a(t) \sin(\omega.t) + b(t) \sin(\omega.t + \varphi) \quad (5)$$

The compounded equation for  $x$  did then yield two coupled differential equations, determining the evolution of the intensities  $a^2$  and  $b^2$ . Their solutions proved that “the simultaneous occurrence of finite stationary oscillations of both the coupling frequencies represents an *unstable* condition and can therefore not be realized in practice”. When the two possible modes of oscillation coexist at a given instant, one of them will spontaneously become more intense at the expense of the other. The “jump” has necessarily to occur for a particular situation, which could also be specified. This was a beautiful result, but we are interested in another property of coupled active and passive oscillators.

We assume that the generator has reached its stationary state of sustained oscillations before it is brought close to the passive oscillator, and we don’t change the natural frequencies  $\omega_1$  and  $\omega_2$ . We simply increase the values of the coupling constants  $k_1$  and  $k_2$ . Assuming that the amplification factor  $\mu \ll 1$ , we expect that harmonics will remain negligible, even for finite values of  $k_1$  and  $k_2$ . The new *stationary state* of the active oscillator should be described by

$$x = A \cos(\omega t) + \dots \quad (6)$$

where the amplitude  $A$  and the (angular) frequency  $\omega$  are unknown. The neglected terms are at best of the order of  $\mu$ . Since equation (4) can be written in the form

$$(1 + k_2) \ddot{y} + v \dot{y} + \omega_2^2 y = k_2 \ddot{x}$$

we see that the passive oscillator will be set in *forced oscillations*. Substituting (6) we get

$$y = -k_2\omega^2 A [F(\omega)\sin(\omega t) + G(\omega)\cos(\omega t)] + \dots \quad (7)$$

where

$$F(\omega) = \frac{v\omega}{(\omega_2^2 - \Omega^2)^2 + (v\omega)^2} \quad \text{and} \quad G(\omega) = \frac{\omega_2^2 - \Omega^2}{(\omega_2^2 - \Omega^2)^2 + (v\omega)^2}$$

For simplicity, we used the notation  $\Omega^2 = (1 + k_2)\omega^2$ . We get thus a typical resonance phenomenon, although we assumed *dynamic coupling*. The function  $F(\omega)$  is always positive and displays a maximum ( $F = 1/v\omega$ ) when  $\Omega = \omega_2$ . At this frequency,  $G(\omega) = 0$ . The forced oscillation is thus the strongest when *ideal resonance* is achieved, but the forced oscillation lags then behind the oscillations of the generator with a phase difference of exactly  $90^\circ$ . At lower frequencies ( $\Omega < \omega_2$ ),  $G(\omega)$  is positive, which means that the passive oscillator tends to remain in step with the active oscillator. Beyond resonance ( $\Omega > \omega_2$ ),  $G(\omega)$  is negative. At very high frequencies, the passive oscillator can't follow very well: forced oscillations have very small amplitudes and they are opposite to those of the generator. The values of  $A$  and  $\omega$  are still unknown, but they will be determined by substituting (6) and (7) in equation (3). The *non-linear* term  $\mu(1 - 2px - x^2)\dot{x}$  is then equal to

$$\mu\omega A[1 - 2pA\cos\omega t - (A^2/2)(1 + \cos 2\omega t)] \sin\omega t = \mu\omega A[1 - (A^2/2) + (A^2/4)] \sin\omega t + \dots$$

To remain consistent, we neglect all terms that oscillate at frequencies  $2\omega$  and  $3\omega$ , since they were also neglected in (6) and (7). Equation (3) yields now the condition

$$(\omega_1^2 - (1 + k_1)\omega^2) \cos\omega t + \mu\omega(1 - a^2) \sin\omega t = k^2\omega^4 [F(\omega)\sin\omega t + G(\omega)\cos\omega t]$$

where  $a = A/2$  and  $k^2 = k_1k_2$ . Since this condition has to be satisfied at every particular instant, we get two relations:

$$a^2 = 1 - \frac{k^2\omega^3}{\mu} F(\omega) \quad \text{and} \quad (1 + k_1)\omega^2 = \omega_1^2 - \frac{k^2\omega^4}{\mu} G(\omega) \quad (8)$$

For the unperturbed generator ( $k_1 = 0$ ), the transistor leads spontaneously to the appearance of stationary oscillations of particular amplitude ( $a = 1$  or  $A = 2$  for the chosen natural units). Of course,  $\omega = \omega_1$ . The values of  $A$  and  $\omega$  will be modified, however, when the active and passive oscillators interact with one another. Figure 4 illustrates the first relation (8).

The amplitude  $A$  of the stationary oscillations can never be increased, since  $F(\omega)$  can't be negative. The reduction of  $A$  with respect to its value for the unperturbed generator has the shape of  $F(\omega)$ , multiplied by  $\omega^3$ . This yields a non-symmetric curve. The highest possible value of  $F(\omega)$  is achieved when  $\Omega^2 = (1 + k_2)\omega^2$ , which implies that  $G(\omega) = 0$ . The second relation (8) shows then that  $(1 + k_1)\omega^2 = \omega_1^2$ . *Ideal resonance* is thus achieved when

$$\omega_2 = g\omega_1 \quad \text{where} \quad g = \sqrt{\frac{1 + k_2}{1 + k_1}} \quad (9)$$

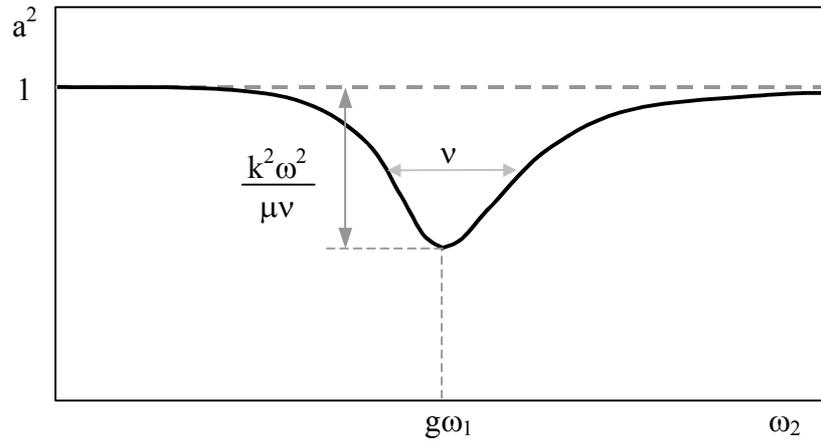


Figure 4: When the active oscillator is coupled to a passive oscillator, the intensity  $a^2$  of the stationary oscillation of the generator reaches its lowest value at *ideal resonance*.

The active oscillator is able to draw a certain amount of energy from the batteries, as a result of the peculiar non-linear characteristics of element T, but it is also able to *transfer some energy* to the passive oscillator. There exists a resonance phenomenon, but the dynamic coupling implies that optimal energy transfer is not necessarily achieved when the natural frequency of the passive oscillator is equal to the natural frequency of the active oscillator. The actual condition (9) depends on the values of the coupling constants  $k_1$  and  $k_2$ . This is very important, since the values of  $\omega_1$  and  $\omega_2$  are fixed. The probe could be tuned in advance to achieve optimal response to a particular pathological state in a particular organ, but there can exist individual variations and the requirement of sharp tuning would then not be desirable at all. Fortunately, we don't need it.

Empirically, it appears that it is useful to move the probe with respect to the tested tissue, with a sort of scanning motion until one finds the position where the spectrum analyser displays the largest "dip" for the stationary oscillations of the active oscillator. This seems to correspond to an adjustment of the values of the coupling constants  $k_1$  and  $k_2$  until condition (9) is satisfied for given values of  $\omega_2$  and  $\omega_1$ . Since  $k_1 = C/C_1$  and  $k_2 = C/C_2$ , we are simply choosing a place where the capacity  $C$  is adequate for ideal resonance. For  $k_1 \ll 1$  and  $k_2 \ll 1$ , we would get  $g = 1$  or eventually,  $g = 1 + (k_2 - k_1)/2$ . Since the values of  $C_1$  and  $C_2$  are very small (to get high values for the natural frequencies  $\omega_1$  and  $\omega_2$ ), we expect that  $k_1$  and  $k_2$  can easily reach relatively large values with respect to 1. This increases the efficiency of the system for the discovery of possible resonance phenomena!

It should be noted that the depth of the "dip" is not only proportional to  $k^2 = k_1 k_2$ , but also to  $\omega^2$ . This is a consequence of the dynamic coupling, but we know<sup>1</sup> that the high frequency domain is also very favourable for the detection of significant differences between malignant and normal tissues. This results from a higher density of bound water molecules or dipolar protein fragments, sticking out of membrane surfaces. They can easily be set in rotation, even if these rotations are partly hindered, but this leads to increased energy absorption. This is also favourable for our purposes, since the depth of the "dip" in figure 4 is proportional to  $1/v = RC_2$ . The total energy of the active and passive oscillators is proportional to

$$x^2 + y^2 = A^2 \cos^2 \omega t + (k_2 \omega^2)^2 A^2 [F(\omega) \sin(\omega t) + G(\omega) \cos(\omega t)]^2$$

The average value is thus proportional to

$$1 + \frac{k_2^2 \omega^4}{(\omega_2^2 - \Omega^2)^2 + (v\omega)^2}$$

We see that the coupling *increases* the total energy drawn from the batteries. It reaches its largest value when ideal resonance is achieved. The relative increase is then equal to  $k_2^2(\omega/v)^2$ . In other words, the active oscillator draws *as much energy it can* from the batteries, to share it with the passive oscillator. Nevertheless, its own amplitude of oscillation is decreased, as indicated by figure 4. The active oscillator seems to make a “big effort”.

Another remarkable feature is that the depth of the “dip” is proportional to  $1/\mu$ . This fact demonstrates most dramatically that *non-linear systems are qualitatively different from linear ones*. The behaviour changes very abruptly, as soon as  $\mu \neq 0$ , but a negative resistance is not sufficient to allow for an energy transfer. There has to be a term in  $x^2$  in equation (3), while the term  $px$  is irrelevant when  $\mu$  is very small.

What are the possible values of the (angular) frequency  $\omega$ ? We know already that *ideal resonance* requires that  $(1 + k_1)\omega^2 = \omega_1^2$  and  $(1 + k_2)\omega^2 = \omega_2^2$ , which leads to (9), but in general, there are two possible frequencies. They are determined by the second relation (8) and the expression of  $G(\omega)$ . Setting  $c_1 = 1 + k_1$ ,  $c_2 = 1 + k_2$ ,  $\Omega_1^2 = \omega_1^2 / c_1$  and  $\Omega_2^2 = \omega_2^2 / c_2$ , we get the equation

$$(\Omega_1^2 - \omega^2) \left( (\Omega_2^2 - \omega^2)^2 + (v\omega/c_2)^2 \right) = k^2 \omega^4 (\Omega_2^2 - \omega^2) / \mu c_1 c_2$$

Without coupling, there are two possible solutions:  $\omega = \omega_1$  for the active oscillator and  $\omega = \omega_2$  for the passive oscillator. For the coupled oscillators, we get a solution  $\omega^2 = \Omega_1^2 + \varepsilon$ , where  $\varepsilon$  is small and can thus be calculated very easily. This yields

$$\omega^2 = \Omega_1^2 + \frac{k^2 \Omega_1^4}{\mu c_1 c_2} \frac{(\Omega_1^2 - \Omega_2^2)}{(\Omega_1^2 - \Omega_2^2)^2 + (v\Omega_2/c_2)^2}$$

The same procedure yields also the solution where  $\omega$  is close to  $\Omega_2$ , but according to van der Pol's reasoning, we expect that the system tends to preserve its previous state. The generator was oscillating at the frequency  $\omega_1$  before it was brought close to the passive oscillator. The frequency  $\omega$  should thus remain close to  $\Omega_1$ . It is increased when  $\omega_2 < g\omega_1$  (implying that  $\Omega_2 < \Omega_1$ ). When  $\omega_2 > g\omega_1$ , it is decreased. Anyway, we don't have to worry about the frequency, since the experimental procedure calls only for the observation of an eventual reduction of the amplitude  $A$ .

### 3. Non-linear resonance interactions at several frequencies

So far, it appeared that the probe is able to “feel” what happens inside biological tissues and to “tell” us that it is transferring energy, since it oscillates with decreased amplitude. Although the passive oscillator remains hidden, we are able to detect a resonance for its forced oscillations. This appeared already when  $\mu \ll 1$  and was even particularly pronounced for very small values of  $\mu$ , but we got then only *one* frequency component. A larger value of  $\mu$  would produce harmonics and this could allow for *a simultaneous search of non-linear resonance interactions on several frequency channels*. This is so advantageous, that we may

prefer the compromise where the effects of non-linear resonance interactions are decreased for the fundamental frequency, but can become apparent for harmonics. In Vedruccio's set-up, one can easily observe *three* spectral lines, corresponding to the fundamental frequency and the two first harmonics. Empirically, he discovered that they have the following properties:

- The first, second and third spectral lines *can decrease independently from one another*, so that one has to accept the possible detection of different resonance phenomena.
- The second and third lines seem to be depressed more easily than the first one.
- A depression of the first line in addition to those of the second and third line indicates a particularly severe pathological modification. This is useful for diagnostic purposes.

Each one of these points requires a theoretical justification. We will thus try to solve equations (3) and (4) for a value of  $\mu$  that is large enough to produce a non negligible second and third harmonic, but small enough to yield a convergent series of higher frequency components. This means that we replace (6) by the more general solution

$$x = A \cos(\omega t) + A_1 \cos(2\omega t) + B_1 \sin(2\omega t) + A_2 \cos(3\omega t) + B_2 \sin(3\omega t) + \dots \quad (10)$$

The amplitudes  $A_1$  and  $B_1$  are proportional to  $\mu$ , while  $A_2$  and  $B_2$  are proportional to  $\mu^2$ , where  $\mu < 1$ . It should be noted that (6) and (7) could have been replaced by

$$x = B \sin(\omega t) + \dots \quad \text{and} \quad y = -k_2 \omega^2 B [G(\omega) \sin(\omega t) - F(\omega) \cos(\omega t)] + \dots$$

This is merely a matter of choosing the instant  $t = 0$ , but when we consider harmonics, we are obliged to allow for possible phase differences. We consider a single passive oscillator, interacting with the active oscillator, but we can imagine several passive oscillators with *different* characteristics, so that one of them can eventually be set in resonance. We have now to introduce (10) in equation (4), but we can immediately assert that this will yield the following response:

$$\begin{aligned} y = & -k_2 \omega^2 A [F(\omega) \sin(\omega t) + G(\omega) \cos(\omega t)] - k_2 \omega^2 A_1 [F(2\omega) \sin(2\omega t) + G(2\omega) \cos(2\omega t)] \\ & - k_2 \omega^2 B_1 [G(2\omega) \sin(2\omega t) - F(2\omega) \cos(2\omega t)] - k_2 \omega^2 A_2 [F(3\omega) \sin(3\omega t) + G(3\omega) \cos(3\omega t)] \\ & - k_2 \omega^2 B_2 [G(3\omega) \sin(3\omega t) - F(3\omega) \cos(3\omega t)] + \dots \end{aligned} \quad (11)$$

To solve (3), we have to evaluate the non-linear term  $-\mu(1-2px - x^2)dx/dt = dZ/dt$ , where  $Z = -\mu[x - px^2 - (1/3)x^3]$ . Since we limit our perturbation calculation to second order effects, it is sufficient to retain only the following terms in  $Z$ :

$$\begin{aligned} x &= A \cos(\omega t) + A_1 \cos(2\omega t) + B_1 \sin(2\omega t) \\ x^2 &= A^2 \cos^2(\omega t) + 2AA_1 \cos(\omega t) \cos(2\omega t) + 2AB_1 \cos(\omega t) \sin(2\omega t) + \dots \\ &= (A^2/2)(1 - \cos 2\omega t) + AA_1(\cos \omega t + \cos 3\omega t) + AB_1(\sin \omega t + \sin 3\omega t) + \dots \\ x^3 &= A^3 \cos^3(\omega t) + 3A^2 A_1 \cos^2(\omega t) \cos(2\omega t) + 3A^2 B_1 \cos^2(\omega t) \sin(2\omega t) + \dots \\ &= (A^3/4)(\cos 3\omega t + 3\cos \omega t) + 3(A^2/2)(A_1 \cos 2\omega t + B_1 \sin 2\omega t) + \dots \end{aligned}$$

Higher frequency terms are neglected, as well as other constant terms, since we have to calculate the time derivative

$$dZ/dt = \mu\omega [C\sin(\omega t) + D\cos(\omega t) + C_1\sin(2\omega t) + D_1\cos(2\omega t) + C_2\sin(3\omega t) + D_2\cos(3\omega t)]$$

Setting again  $A = 2a$ , the coefficients are

$$\begin{aligned} C &= A(1 - a^2 - pA_1) & D &= -pAB_1 \\ C_1 &= (2 - A^2)A_1 - pA^2 & D_1 &= -(1 - A^2)B_1 \\ C_2 &= -A(a^2 + 3pA_1) & D_2 &= 3pAB_1 \end{aligned}$$

Substituting (10), (11) and  $dZ/dt$  in equation (3), we get an ensemble of conditions that have to be satisfied at every particular instant:

$$(\omega_1^2 - (1 + k_1)\omega^2) A + \mu\omega D = k^2\omega^4 A G(\omega) \quad (12)$$

$$\mu\omega C = k^2\omega^4 A F(\omega) \quad (13)$$

$$(\omega_1^2 - (1 + k_1)4\omega^2) A_1 + \mu\omega D_1 = k^2\omega^4 [G(2\omega)A_1 - F(2\omega)B_1] \quad (14)$$

$$(\omega_1^2 - (1 + k_1)4\omega^2) B_1 + \mu\omega C_1 = k^2\omega^4 [F(2\omega)A_1 + G(2\omega)B_1] \quad (15)$$

$$(\omega_1^2 - (1 + k_1)9\omega^2) A_2 + \mu\omega D_2 = k^2\omega^4 [G(3\omega)A_2 - F(3\omega)B_2] \quad (16)$$

$$(\omega_1^2 - (1 + k_1)9\omega^2) B_2 + \mu\omega C_2 = k^2\omega^4 [F(3\omega)A_2 + G(3\omega)B_2] \quad (17)$$

In (12) and (13), we can drop the common factor  $A$ . This yields the result (8), with a *first* order correction for  $a = A/2$  and a *second* order correction for  $\omega$ :

$$a^2 = 1 - (k^2\omega^2/\mu)F(\omega) - pA_1 \quad \text{and} \quad (1 + k_1)\omega^2 = \omega_1^2 - k^2\omega^4 G(\omega) + \mu\omega pB_1 \quad (18)$$

Solving (14) and (15), we get for the lowest order expressions,

$$A_1 = \frac{\mu p \omega (k^2 \omega^4 F(2\omega)) A^2}{(\omega_1^2 - (1 + k_1)4\omega^2 - k^2 \omega^4 G(2\omega))^2 + (k^2 \omega^4 F(2\omega))^2}$$

$$B_1 = \frac{\mu p \omega A^2 (\omega_1^2 - (1 + k_1)4\omega^2 - k^2 \omega^4 G(2\omega))}{(\omega_1^2 - (1 + k_1)4\omega^2 - k^2 \omega^4 G(2\omega))^2 + (k^2 \omega^4 F(2\omega))^2}$$

They were obtained by adopting the lowest order approximation of (18), so that  $a = 1$  and  $(1 + k_1)\omega^2 = \omega_1^2$ . We see that  $A_1 = B_1 = 0$  when  $p = 0$ . There would thus be no second spectral line, if we had adopted the usual van der Pol equation. We set now

$$\omega = q\omega_1, \quad \text{where} \quad q^2 = 1/(1 + k_1)$$

Thus,

$$A_1 = 0 \quad \text{and} \quad B_1 = -\frac{\mu p q A^2}{3\omega_1} \quad \text{when} \quad k = 0$$

This is equivalent to the result obtained by van der Pol for the unperturbed generator<sup>16</sup>, but he used a model where  $q = 1$ . His perturbation calculation was very efficient, but less transparent and he assumed *a priori* that  $A_1 = 0$ . This is not true anymore for coupled

oscillators.  $A_1$  reaches even its largest value when  $F(2\omega)$  is maximum. Ideal resonance is then achieved when

$$(1 + k_2)4\omega^2 = \omega_2^2 \quad \text{and} \quad (1 + k_1)\omega^2 = \omega_1^2 \quad \text{or} \quad \omega_2 = 2g\omega_1$$

For small values of the coupling constants, this condition reduces to  $\omega_2 = 2\omega_1$ , but we see again that *ideal resonance* could be achieved or at least, approached more easily by simply adjusting the coupling. Setting  $c = \mu p q A^2 / \omega_1$  and  $z = k^2 \omega^2 F(2\omega) = k^2 \omega / 2\nu$  at resonance, where  $G(2\omega) = 0$ , we see also that the magnitude of the second spectral line will be determined by

$$A_1 = \frac{c q^2 z}{9 + (q^2 z)^2} \quad \text{and} \quad B_1 = \frac{-3c}{9 + (q^2 z)^2}$$

The absolute value of  $B_1$  decreases for increasing values of  $z$  or  $k^2$ , but  $A_1$  can increase. Does this imply that the height of the second spectral line could eventually be increased? No, since the intensity of the first harmonic wave component is proportional to

$$A_1^2 + B_2^2 = \frac{c^2}{9 + (q^2 z)^2}$$

This expression can be approximated by  $(c/3)^2(1 - q^4 k^4 \omega^4 [F(2\omega)]^2 / 18)$ . It appears therefore that an ideal resonance (for  $\omega_2 = 2g\omega_1$ ) would *strongly* reduce the second spectral line, especially at high frequencies. According to (17), the intensity of the first spectral line is proportional to

$$A^2 = 4(1 - k^2 \omega^2 F(\omega) / \mu - p A_1)$$

It is not so strongly affected by  $F(\omega)$  when  $\mu$  is not exceedingly small, but we see that  $A_1$ , which increases for relatively small values of  $z$ , could also contribute to a reduction of  $A^2$ . This is an example of *lateral interactions* between neighbouring spectral lines.

Solving (16) and (17), we get for ideal resonance ( $\omega_2 = 3g\omega_1$ ),

$$A_2 = \frac{3\mu\omega p A B_1 (8\omega_1^2)^2 - \mu\omega A a^2 k^2 \omega^4 F(3\omega)}{(8\omega_1^2)^2 + (k^2 \omega^4 F(3\omega))^2} = \frac{3p\mu p q A B_1}{8\omega_1} \quad \text{when } k = 0$$

$$B_2 = \frac{-\mu\omega A a^2 (8\omega_1^2)^2}{(8\omega_1^2)^2 + (k^2 \omega^4 F(3\omega))^2} = \frac{-\mu q A^3}{32\omega_1} \quad \text{when } k = 0$$

The particular case of the *unperturbed generator* ( $k = 0$ ) corresponds to van der Pol's result<sup>14</sup> (where  $g = 1$ ), with a sign correction for  $B_2$ . The cause of this error can be traced in his calculations. We see now that even for the unperturbed generator, the intensity of the third spectral line is influenced by the existence of a second spectral line ( $A_2$  is proportional to  $B_1$ ). We also see that a coupling with a passive oscillator that resonates when  $\omega_2 = 3g\omega_1$  will reduce the intensity of the third spectral line for two cooperative reasons:  $F(3\omega)$  appears not only in both denominators, but also in the numerator of  $A_2$ . These effects are more

pronounced at high frequencies, where malignant tissues have larger conductivities and absorb thus more energy from the generator.

Because of the non-linear term in equation (3), larger values of  $\mu$  would lead to more *combination frequencies* and therefore to more harmonics and increased lateral interactions, even beyond the first neighbour. Analytical calculations would become more complicated, but we understand already the underlying processes. The parameter  $\mu$  should thus be large enough to allow for simultaneous explorations on different frequency channels, but not too large, to limit confusing lateral interactions between neighbouring spectral lines.

#### 4. Discussion and Conclusions

The efficiency of Vedruccio's EM cancer detector results from a favourable conjunction of two elements. (1) *EM waves can stimulate minute electrical oscillations in biological tissues* and they allow for resonance effects that depend on the pathological state of these tissues. (2) By coupling the oscillations of the probe with those that can appear inside biological tissues we get the highly remarkable phenomenon of "non-linear resonance interaction". It has been analysed in detail, but it may be useful to summarise the essential steps, especially for the non-mathematically minded reader.

- The probe contains *an auto-oscillator*, generating stationary oscillations of preset, but tuneable amplitude and repetition frequency. It is possible to get several harmonics in addition to the fundamental frequency, and this allows for *a simultaneous search of possible resonance interactions on several frequency channels*.
- It is sufficient to bring the probe near the biological tissue that should be tested. The generator is then able to "feel" the hidden response of passive oscillators inside the biological tissue and to "tell" us the result by means of *non-linear resonance interactions*. They result from the fact that the generator draws energy from the batteries in such a way that we get stationary oscillations of predetermined amplitude, but this amplitude will be *reduced when the generator transfers some of its energy to a passive oscillator*. Thus, we can detect a resonance at any one of the available frequencies, through the appearance of a "dip" for the corresponding spectral line.
- The coupling is achieved in a special way, since the probe contains an antenna, behaving (in the near field) like an open capacity. The oscillating electric field acts there on charged particles, hidden inside the tested biological material. Ions can oscillate, cell membranes can be polarized and small dipolar particles can be set in rotation, but these processes are modified by malignant alterations of biological tissues. In spite of fixed frequencies for the active and passive oscillators, it is possible to realize or approach *ideal resonance* by adjusting the distance between the probe and the tested tissue.
- Since the required intensities of the EM wave are very low, this new and surprising method of cancer detection implies no health hazard. It is *user-friendly*, as well for the patient as for the medical doctor. The actual relation between specific pathological conditions and the observed, differentiated reductions of the three first spectral lines has to be established, of course, by detailed medical testing.

The differential equations (3) and (4) can easily be solved by numerical integration. This allows for a flexible exploration of the role of all parameters and for further generalizations, but the results have to be interpreted in terms of the analytical treatment we presented here.

Clarbruno Vedruccio observed that when two probes that have nearly identical natural frequencies are put side by side, they oscillate at exactly the same frequency, although their

natural frequencies may differ by about 10 MHz. The spectrum analyser shows superposed spectral lines, as if there did exist only one, but stronger generator. This observation confirms the validity of the proposed theory, since such a “synchronization” is known to occur for coupled van der Pol oscillators<sup>12,13</sup>.

The interaction between a non-linear active oscillator and an ordinary (linear) passive oscillator leads to the peculiar phenomenon of “non-linear resonance interaction”. A similar behaviour is known for a *grid-dip meter*. Initially, it contained a triode<sup>17</sup> that was associated with an oscillating circuit in such a way that it delivered a stationary oscillation at *one* particular, easily tuneable frequency. There was no antenna and no emitted wave, but the active oscillator could be coupled by *magnetic induction* with another oscillating circuit, containing a real coil. When such a grid-dip meter is tuned, so that its natural frequency is identical to the natural frequency of the passive oscillator, there will be a resonance. Since the active oscillator is transferring energy to the passive oscillator, the oscillating current passing through the self of the active oscillator is reduced, and a measuring instrument, included in the grid circuit, will indicate this effect. At resonance, there appears a “grid-dip”, but to avoid ambiguities, the active generator should produce no harmonics.

The EM cancer detector is different, since it allows for an *electric* coupling, activating charged particles inside biological tissues or other polarizable materials. Moreover, there are *harmonics*, but the spectrum analyser allows for a distinction of possible resonance effects for anyone of the frequency components. A simple grid-dip meter has been used, however, in a nice experiment, for didactic purposes<sup>18</sup>. Since Co<sup>59</sup> nuclei have a magnetic moment that undergoes Larmor precession in a given magnetic field, it is possible to stimulate this precession by applying an adequate radiofrequency. When a grid-dip meter provides this signal, one gets a small dip at 213.1 MHz, which is the resonance frequency for Co<sup>59</sup>, since cobalt powder provides a sufficiently strong response for detection by this simple method.

C. Vedruccio called his electronic system a “*bioscanner*”. The Italian company Galileo Avionica produces this instrument and the associated software under the trade name *TRIMprob* (Tissue Resonance InterferoMeter Probe). The term *interference* should not be understood, however, in the traditional sense of a superposition of signals that can be antagonistic or cooperative. It has to be taken in the more general sense of *a competitive interaction*, since now it is clear that *the available energy is shared* in such a way that the amplitude of the stationary oscillations of the active oscillator are reduced, although the total energy is increased. The active and passive oscillators constitute a new entity, so that it is sufficient to observe the behaviour of one partner to get information about the other partner. This is analogous to the discovery of invisible *extra-solar planets*, by observing the motion of their stars, since they are both rotating about a common centre of mass. This follows from (linear) Newtonian mechanics, however, while the surprising properties of the bioscanner result from its *non-linear* properties.

We hope that this phenomenon will really be able to contribute to early cancer detection, for the benefit of humanity.

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